PHYSICAL MEASUREMENT.

MOORE.

INSTRUMENTS AND METHODS

OF

PHYSICAL MEASUREMENT.

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PREFACE.

THE following pages have been arranged for use in the physical laboratory. The aim has been to be as concise as is consistent with clearness. Many sources of information have been used, and in many cases, perhaps, the *ipsissima verba* of well-known authors.

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Lafayette College,
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INSTRUMENTS USED IN THE MEASUREMENT OF A. LENGTH.

I. The Diagonal Scale.

1. The diagonal scale was formerly extensively used for measuring small distances but modern instruments are more accurate and it has fallen into disuse. It may, however, be employed where great accuracy is not required.

2. *The Principle of Construction.*—Take one division ab of the scale represented in Fig. 1 and divide it into ten equal parts. From O at the top of the scale, draw a diagonal line OC to the first point of division, and then draw lines from the succeeding points of division parallel to OC. The square $Oabf$ will thus be divided into tenths on each side. Take a point marked e_7. Since the triangles Od_7e_7 and OaC are similar,

$$Od_7 : Oa :: d_7e_7 : aC$$

$$d_7e_7 = \frac{Od_7 \times aC}{Oa}$$

Hence, if ab be an inch, $aC = \frac{1}{10}$ inch, and $d_7e_7 = \frac{Od_7}{Oa} \cdot \frac{1}{10} = \frac{7}{10} \cdot \frac{1}{10} = \frac{7}{100} = .07$ inch.

Similarly, $d_4e_4 = \frac{4}{100} = .04$; also, $d_1e_1 = .01$, $d_2e_2 = .02$, $d_3e_3 = .03$, etc. The point n is $\frac{3}{10} + \frac{6}{100}$ of an inch from Oa, hence .36.

3. *How to Read the Scale.*—Hence, to read the scale measure the integral parts on the main scale, the tenths by noting the number of the diagonal line and the hundredths by observing the number of the horizontal line which intersects the diagonal line.

In measuring the distance with a dividers, be careful to keep both points on the same horizontal line.

II. The Vernier.

4. The Vernier, named after its inventor, 1631, is an instrument for measuring small distances.

The Principle of Construction of Straight Verniers—

5. *a. Direct Verniers.*—Take any scale of equal parts, one divided into tenths of inches (for example, Fig. 2). Lay along its edge a strip of cardboard upon which the vernier is to be constructed. Lay off on the cardboard the length of nine divisions of the scale, but divide this distance into ten equal parts. Then ten divisions of the vernier $= 9$ divisions of the scale $= \frac{9}{10}$ inch, and one division of the vernier $= \frac{9}{100}$ inch.

Hence, each division of the scale is greater than each division of the vernier by $\frac{1}{100}$ inch, for $\frac{10}{100} - \frac{9}{100} = \frac{1}{100}$. Hence, differences between lines can be measured to hundredths of an inch.

6. *b. Retrogade Verniers.*—It is obvious that instead of making 10 divisions of the vernier equal to 9 divisions of the scale, 11 scale divisions might have been made equal to 10 of the vernier, then

10 divisions of vernier $= 11$ divisions of scale $= \frac{11}{10}$ inch.

1 division of vernier $= \frac{11}{100}$ inch.

Hence, each division of the scale is less than each division of the vernier by $\frac{1}{100}$ inch, for $\frac{11}{100} - \frac{10}{100} = \frac{1}{100}$. In this case the numbers of the vernier (Fig. 3) and scale will run in opposite directions.

7. *Metric Verniers*—either direct or retrogade—are constructed on the same principle. Suppose, for example (Fig. 4), that each scale division is one centimeter. Let 10 divisions of the vernier $= 9$ scale divisions $= 9 \times 1$ centimeter $= 9 \times \frac{1}{100} m = \frac{9}{100} m$. Then one vernier division $= \frac{1}{10}$ of $\frac{9}{100} m. = \frac{9}{1000} m.$ The difference between the vernier and scale divisions is, therefore, $\frac{10}{1000} - \frac{9}{1000} = \frac{1}{1000} m = 1 mm.$

8. *General Algebraic Expression of the Principle.*—Let a be the value of the scale divisions; let n be the number of scale divisions and $n \pm 1$ the number of divisions of the vernier. Then $(n \pm 1)$ divisions of the vernier $= n$, each one having the value of the scale division, or $n \cdot a$. One division of the vernier $= \frac{n}{n \pm 1} a$. The difference between the divisions of the vernier and scale is $a - \frac{n}{n \pm 1} a = \frac{a}{n \pm 1}$.

Hence to find to *what any vernier will read, divide the value of the scale divisions by the number of divisions of the vernier.*

9. *How to Read a Vernier.—Read the scale for the larger divisions by observing the position of the vernier zero and the coinciding lines of the vernier and scale for the smaller fractional parts.*

10. *Special Case.*—In the English Barometer Scale, the scale is divided into $\frac{1}{20}$ of an inch, and 25 vernier divisions $= 24 \times \frac{1}{20} = \frac{24}{20}$ of an inch. Hence, one vernier division $= \frac{1}{25}$ of $\frac{24}{20}$ inch $= \frac{24}{500} = .045$. The difference $.050 - .048 = .002$ inch.

11. *How to read the Vernier when no lines exactly coincide.—The usual practice is "to take the mean of the readings which would be given by a coincidence of either pair of bounding lines."*

III. Mayer's Vernier Microscope.

12. In this instrument a finely divided scale is placed on the stage of the microscope and a glass vernier in the eye-piece.—*Sci. Am. Sup.*, 1877, p. 1136.

IV. The Vernier Caliper.

13. The Vernier caliper (Fig. 5) consists of a graduated metal bar, having a jaw at the end at right angles to the axis of the bar. A moveable jaw at right angles to another bar is made to slide freely alongside the edge of the graduated bar. This second bar contains a vernier. The sliding part of the caliper can be clamped to the main bar by two thumb-screws, one of which fastens the second jaw to it, and the other is intended for a clamp which has a tangent screw, the object of which is to make small adjustments after the instrument has been roughly set.

14. *The Graduation.*—On one side the bar is divided into inches and fiftieths. The vernier is so constructed that 20 of its divisions equal 19 of the scale; hence, $20\,V = \frac{19}{50}$; hence, one vernier division $= \frac{19}{1000}$, and the difference between the lengths of the divisions of the vernier and scale is $\frac{1}{1000}$ inch.

On the other side the limb is divided into millimeters and with the vernier will read to the fiftieth of a millimeter.

15. *What Kind of Measurements May be Made.*—If a short rod or bar is placed between the jaws of the calipers and the set-screw S_1 fixed, further adjustment may be made with the tangent screw T, after which the length of the rod may be read off the scale and vernier.

The jaws are rounded on the outside so that they may be inserted into tubes; hence, the diameter of a hollow space may be measured.

Sometimes there is a mark placed on the scale for inside and outside measurements. If not, the width of the jaws must be added to the scale reading. For the Brown & Sharpe Caliper this diameter, when the jaws are closed, is .250 in.

V. The Beam Compass.

16. Fig. 6 consists of a graduated bar upon which two clamps holding two points may be made to slide. A tangent screw is attached to one of the clamps for fine adjustments. The beam compass may be used for measuring the length of lines where the extremities are slightly inaccessible or at a considerable distance apart. It may also be used to verify the divisions of a scale by comparison with the one on the bar or with a standard. The method of using it is obvious.

VI. The Kathetometer.

17. The Kathetometer (Fig. 7), as its name indicates, is an instrument used to measure vertical distances. It consists essentially of a vertical scale to which is attached a plate with a vernier free to slide along the scale, and a telescope and level at right angles to the plate. The whole is so supported that it may be made to rotate around an upright support. The idea involved in the construction of the instrument is such that (1) *the scale must be vertical* and (2) *the axis of the telescope at right angles to the vertical.*

18. *How to Use the Instrument.*—The cross-wires of the telescope are focussed upon one end of a vertical line, and the scale and vernier read; the sliding part is then moved so that the cross wires will coincide with the other end of the line and the reading taken. The difference between these readings is the required distance.

19. *First Adjustment.—Place the standard in a vertical position.*—Let A, B, C (Fig. 7*a*) be the three screws forming the feet of the kathetometer. Rotate the instrument on its standard so that the level on top of the telescope is parallel to a line connecting A and B. Turn B up or down until the bubble occupies its middle position. Turn the instrument through 180° and if the bubble is not in its middle position screw A up or down for half the distance and complete the adjustment by using the screw at one end of the level. Repeat these operations until the bubble remains stationary whenever the level is reversed on AB.

Turn the instrument now until the axis of the level is parallel to a line perpendicular to AB, adjust the third screw C; repeat these operations until the bubble is at the centre in all positions. The standard is now vertical.

20. *Second Adjustment.—Adjust the telescope for error of parallax.*—The error of parallax is that due to the noncoincidence of the plane of the image of the distant object with the plane containing the image of the cross-wires. It may be detected by focussing the telescope upon a distant object and at the same time getting a distinct image of the cross-wires by adjusting the sliding-tube of the eye-glass. If the image of the distant object is in front of the cross-wires and the head is moved to and fro or up or down the image will seem to move in the opposite direction. If the image is between the eye and the plane of the cross-wires when the head is moved the apparent motions will be in the same direction. The obvious method of correcting this error is to focus the telescope on a distant scale and vary the positions of the images of the cross-wires and distant scale until the required coincidence has been established which can be determined by the fact that there is no apparent motion of the image when the head is moved up or down and to or fro.

21. *Third Adjustment.—Adjust the telescope for error of collimation.*—The error of collimation is that due to the non-coincidence of the point of intersection of the cross-wires with the optic axis of the lenses.

The error is detected by rotating the telescope in its Y's when focussed upon a distant object (Fig. 7*c*). The image of the distant object will appear to rotate around the intersection of the cross-wires. The cross-wires are attached to a brass ring which may be centered by four screws placed upon the circumference of the tube of the telescope 90° from each other. To effect the necessary adjustment sight a distant vertical scale placing the horizontal cross-wire upon one of the lines. Rotate the telescope through 180° and if there is not coincidence screw in or out the vertical screw half the distance of the displacement of the wire. Rotate back to its original position and if there is coincidence the correction has been made for the horizontal wire. Subject the other wire to the same treatment.

22. *Fourth Adjustment.—Place the line of collimation perpendicular to the vertical axis of the kathetometer.*—Sight the telescope (Fig. 7*d*) at a distant scale making the intersection of the cross-wires coincide with one of the lines. Now rotate the whole instrument through 180°. Remove the telescope from its Y's and replace it in a reversed position. If the intersection of the cross-wires still coincides with the line the adjustment is already effected. If not, an examination of the instrument will show that the telescope support may be slightly rotated about a horizontal axis. Make the adjustment for half the distance, the intersection fails to coincide. Repeat the operation until on reversing the whole instrument and the telescope coincidence remains established. The level which has been thrown out of adjustment by rotating the telescope may now be rectified by using its screw and the instrument is ready for use.

Test the scale by setting up a standard scale in a vertical plane and comparing the kathetometer scale with it.

VII. The Reading Telescope.

23. Sometimes a scale may be placed by the side of a vertical distance to be measured and a telescope (Fig. 8) having an up and down motion on a vertical rod be used for reading. Cross-wires in the eye-piece are used to accurately locate the extremities of the line representing the distances.

VIII. Stage Micrometer with Camera Lucida.

24. A scale (Fig. 9) divided on glass to $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{2000}$ of an inch, or $\frac{1}{10}^{mm}$ or $\frac{1}{100}^{mm}$ is placed upon the stage of a compound microscope.

The instrument is tilted until the tube occupies a horizontal position. A camera lucida is attached to the eye-piece when by looking through it on the table, which ought to be distant 10 inches, an enlarged image of the scale becomes visible. This scale may be marked upon a piece of paper. The scale may now be removed from the stage and a small object substituted. The image of the object will be projected upon the scale upon the paper and the exact dimensions read off just as one reads from a two-foot rule.

The visibility of the lines is increased by oblique illumination and small diaphragm.

IX. Jackson's Eye-Piece Micrometer.

25. A small glass plate (Fig. 10) having a scale of equal parts engraved upon it is placed between the two lenses of the negative eye-piece of a compound microscope. This scale can be moved across the field by a micrometer screw.

26. *How to Determine the Value of the Divisions of the Eye-piece Scale.*—A glass scale is placed upon the stage of the microscope, the value of whose divisions is known, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$ of an inch, or $\frac{1}{100}$, $\frac{1}{1000}$ mm. Having carefully focussed the instrument, observe the number of divisions of the eye-piece scale exactly equal to a certain number of divisions of the stage scale. This number will vary for different powers. If the microscope has a draw tube coincidence of lines can be made exact by drawing the tube in or out. The value of a single division is then exactly obtained.

X. Prof. Quincke's Kathetometer Microscope

27. (Fig. 11) consists of a microscope mounted on a pair of Y's having a glass base. This glass base slides upon a support which may be leveled and raised and lowered. The top of the support is glass so that there is great freedom of motion of the microscope. The eye-piece contains a scale of equal parts of known value.

The ordinary **Kathetometer Microscope** (Fig. 12) is the same as the above, except that the microscope is mounted upon a vertical support and may be moved up and down. It is used in the same manner as the kathetometer.

XI. The Screw.

28. *The Principle of Construction.*—(1) The nut may be fixed and the screw be allowed to advance (Fig. 13), or (2) the screw may be prevented from advancing while the nut moves in a direction parallel to the axis of the screw between guides which prevent rotation (Fig. 14). It is evident from the construction of the screw that if the head makes one rotation the screw will advance or retreat a distance equal to the pitch of the screw or the distance between the threads. If the head is divided into 100 equal parts and the pitch is $\frac{1}{100}$ of an inch or there are 100 threads to the inch one rotation of the head is equivalent to $\frac{1}{100}$ of an inch; $\frac{1}{100}$ of a rotation of the head is equivalent to $\frac{1}{10000}$ inch. An index is usually placed parallel to the axis of the screw to measure the whole rotations while the parts are measured on the head itself. The index is graduated into divisions equal to the pitch of the screw. If the rotations and fractional parts are multiplied by the pitch the product will give the distance of advance or retreat of the screw. The accuracy of measurement with the screw depends upon—

1. The accuracy of the screw.
2. The permanency in the form of the instrument during measurement.
3. The reproduction of the same degree of pressure on the abutting point.

Three (3) may be effected by using (*a*) the contact lever (Fig. 15), (*b*) the contact level (Fig. 16), (*c*) the electric contact (Fig. 17).

Instruments in Which the Principle of the Screw is Adopted.

29. *A.* **The Micrometer Calipers.**—It is obvious that when the head of the screw (Fig. 18) is turned once the screw advances a distance equal to the distance between its threads. Suppose this distance is the $\frac{1}{40}$ of an inch. Let the head be so large that it may be divided into 25 equal parts. If the head makes one rotation the screw advances one-fortieth of an inch or .025 inch; if it is rotated only $\frac{1}{25}$ of a circumference the screw will advance $\frac{1}{25}$ of $\frac{1}{40} = \frac{1}{1000} = .001$ inch. Suppose, for example, the head was rotated

three turns and twenty divisions of the head, the advance will be .075 + .020 = .095 inch. By placing an object between the end of the screw and the stop (s) and turning the head until contact is made very small dimensions may be measured; hence the name micrometer.

30. *B.* **The Spherometer.**—The *spherometer*, as its name indicates, is an instrument nsed to measure the radius of curvature of a sphere. It was invented by De la Rive. It consists of a micrometer screw placed in the centre of a stand having three legs as represented in Fig. 19.

31. *The Principle of Construction* is the same as that of the micrometer. In Fig. 20 let the circle represent the section of a sphere through its centre. It is obvious that if AB can be measured the radius of the sphere can be obtained. Let ρ be the radius of the sphere, then

$$AB : BD :: BD : BE.$$

$$h : a :: a : AE - AB = 2\rho - h.$$

$$(2\rho - h)\, h = a^2.$$

$$2\rho h - h^2 = a^2.$$

$$2\rho h = a^2 + h^2.$$

$$2\rho = \frac{a^2}{h} + h.$$

$$\rho = \frac{a^2}{2h} + \frac{h}{2}$$

32. *Use of the Instrument.*—The instrument enables us to measure h as follows: First, place it on a perfectly plane piece of glass and screw down the screw until it and the points of the three feet are in the same plane. Contact may be determined by observing (1) if the point of the screw is too low the instrument is unsteady; (2) if the pressure is equal on the three legs friction of the nut will make the whole instrument revolve, or (3) if the instrument is gently pushed over the glass the sound changes in character when the four points are all in contact with the glass, or (4) if the glass and instrument are placed on a sounding box and the

box tapped the rattle will disappear when the screw is down. The instrument is now set at the zero. If the screw is now turned up and the instrument placed on a lens and then screwed down until contact is again established and a reading taken the difference between this reading and the last will give the required value of h.

To get a, set the instrument on a piece of paper and mark the imprint of the points. Now measure the distance from each leg to the axis and take the mean; also measure the distances of the three legs from each other, take the mean and calculate the radius of the circumscribed circle.

Substitute these values in the equation for ρ and its value will be obtained on solving.

It is obvious that if ρ is given *the principal focal distance* of a *lens* may be determined from the formula

$$\frac{1}{f} = (n - 1)\frac{1}{R} + \frac{1}{R'}$$

$n = 1.53$ for common glass.

It is evident that *the thickness of a thin plate* may also be measured by the spherometer, also *the diameters of fibers* of silk, wool, hair, etc.

33. *Defect of the Instrument.*—With the ordinary instrument it is impossible to measure ρ for very small lenses. This has been remedied by an attachment made by Prof. Mayer.—*Sci. Am. Sup.*, Mar. 10, 1877, p. 976.

34. *C.* **The Micrometer Microscope or the Reading Microscope.**—Fig. 21 is a representation of this instrument. It consists of a microscope with wires placed across a hole in a rectangular piece of brass which may be slid across the field by means of a micrometer screw placed upon the side of the tube. The cross-wires are at an angle of 30° to each other and their image is seen directly over the image of the object examined by the microscope.

35. *Principle of Construction.*—Suppose the microscope is focussed on a scale the divisions of which are the $\frac{1}{100}$ inch. Suppose also that the microscope magnifies this distance to one inch. Imagine that the screw has 100 threads to the inch. Then one rotation of the screw is equivalent to $\frac{1}{100}$ inch; but it would take 100 rotations

to move the cross-wires over the magnified inch; hence each rotation is equivalent to $\frac{1}{100}$ of $\frac{1}{100}$ in., or $\frac{1}{10000}$ in. But if the head of the screw is divided into 100 parts, then each part of a rotation will equal $\frac{1}{100}$ of $\frac{1}{10000}$ or $\frac{1}{1,000,000}$ inch.

36. *It is obvious that if we wish to measure small distances with this instrument it is necessary to know exactly the value of each rotation of the head of the screw.* To determine this a finely divided scale of known value is necessary. (1) If the intersection of the cross-wires is focussed on a line at right angles to the scale divisions, (2) and the number of turns of the head observed while the intersection is passing over a known distance on the scale the value of one turn may be determined by dividing this known distance by the number of turns. The cross-wire should be made to travel over various parts of the scale and the motion of the screw should always be in the same direction. Having determined the value of a rotation of the head the micrometer may be employed in many experiments *e.g.* to determine the diameter of a capillary tube, the lengths of various objects, the error of line measure, end measure, etc.

37. *d.* **The Dividing Engine.**—The micrometer screw is applied to the purpose of dividing a straight line into equal parts by means of the dividing engine (Fig. 22). It consists of two parts, a table upon which the body to be divided may be clamped, which table is made to move by a screw parallel to its axis and a cutting tool which has a motion in a vertical plane at right angles to the axis of the screw. The accuracy of the engine depends upon the accuracy of the screw and the permanence of the vertical plane of the cutter.

It is obvious that the dividing engine may be used to measure distances.

To Divide a Line into Equal Parts.

38. 1. **By Means of the Beam Compass.**—Place an accurately divided steel scale so that its length is the prolongation of the line to be divided. Let the beam compass be three or four feet long. Then place one leg of the compass upon a division of the scale and make a mark with the other point. Place the first point on the next division of the scale and mark off another line with the other point. Continue the operation until the scale is copied. The

beam is made long so that the lines shall appear to be straight instead of arcs of circles. Considerable rapidity may be attained if two persons perform these operations, one placing the point on the division of the steel scale and the other drawing the required line with the other point.

To have every fifth line longer than the others a gauge may be placed upon the length to be divided.

To Divide a Line "Originally" into Equal Parts.

39. 1. **By Means of Spring Dividers or Beam Compass.**—Let *ab* be the line to be divided (Fig. 23). From *a* as a center with radius equal to almost half the length describe an arc, from the other end *b* of the line describe an arc with the same radius. Then by means of the eye and magnifying glass divide *cd* into two equal parts. Continue the operation on the half line and then on the quarter, etc., or until the line is divided. The objection to this method is that a line can only be divided into an even number of parts.

40. 2. **By Means of Spring Dividers and a Straight Edge.**—As before (Fig. 24), let *ab* be the line to be divided. From *a* draw a line *ac* of indefinite length making an acute angle with *ab*. Take any convenient length with the dividers and step off the required number of divisions on *ac*. Then join the last division on *ac* with *b* and through each point of division on *ac* draw a line parallel to *cb*. The parallel lines will cut *ab* so that the divisions will be equal.

41. 3. **Another Method.**—Let *ab* (Fig. 25) be the given line. Through the extremity *a* draw a line making an acute angle with *ab*: through *b* draw a line *bo* parallel to *ac*. With the spring dividers step off any convenient distance the same number of times as are required on *ab*. Similarly step off the same distance from *b* on *bd*. Join the corresponding points on the two lines and the parallel lines will divide the line *ab* as required.

B. ANGLES.

I. Arc Verniers.

42. The *principle applied in the construction* of straight verniers may be applied to the making of arc verniers (Fig. 26). It is customary to divide circular scales into 60′, 30′, 20′, 15′, 10′.

43. *A. Let the scale be divided into degrees*—60′ *e.g.*

1. Then if 6 vernier divisions are made equal to 5 scale divisions, the vernier will read to 10′.

$6V = 5 \cdot 60' = 300'.$

$1V = 50'.$

Hence the difference is $60' - 50' = 10'$.

2. If 10 vernier divisions equal 9 scale divisions the reading will be to 6′.
3. If 12 vernier divisions equal 11 scale divisions the vernier will read to 5′.
4. If 20 vernier divisions equal 19 scale divisions the vernier will read to 3′.
5. If 60 vernier divisions equal 59 scale divisions, the vernier will read to 1′.

44. *B. Let the scale be divided into* $\frac{1}{2}° = 30'$.

1. If 30 vernier divisions equal 29 scale divisions the vernier will read to 1′.
2. If 30 vernier divisions equal 31 scale divisions the vernier will read to 1′.
3. If 15 vernier divisions equal 16 scale divisions the vernier will read to 2′.

45. *C. Let the scale be divided to* 20′ (*usual method*).

1. If $20V = 19S$, the vernier will read to 1′.
2. If $40V = 41S$, the vernier will read to 30″.
3. If $60V = 59S$, the vernier will equal 20″.
4. If $20V = 21S$, the vernier will equal 1′. This is the usual relation.

46. *D. Let the scale be divided to* 15′.

1. If $60V = 59S$, the vernier will read to 15″.

47. *E. Let the scale be divided to* 10′.

1. If $60V = 59S$, the vernier will read to 10″.

48. *The error due to eccentricity in mounting the arc* may be obviated by using two verniers 180° apart. Suppose (Fig. 26*a*) one vernier is at V instead of at V_1', and the other at V_2 instead of V_2'.

Let x = the true reading of the vernier.
Let e = the error of the vernier.
Let R_1 = the reading of the upper vernier.
Let R_2 = the reading of the lower vernier.

Then $x = R_1 + e$ for upper vernier and $x + 180 = R_2 - e$ for the lower vernier; $2x + 180 = R_1 + R_2$; $x = \frac{R_1 + (R_2 - 180)}{2}$;

Hence, the true reading is the mean of the reading of the upper vernier and the reading of the lower vernier less 180°.

II. The Spirit Level.

49. The *Spirit Level* (Fig. 27) consists of a glass tube closed at both ends containing a mobile liquid like alcohol or ether. The tube is filled excepting a small air bubble. The tube is not straight but curved in an arc of a circle of very great radius. The tube is placed upon a brass plate the surface of which should be parallel to the axis of the tube. Striding levels and hanging levels are but modifications of the common level.

The glass above the air bubble is graduated. The object of the divisions is (1) to determine the middle point, (2) the horizontal plane and (3) to measure small angles of inclination.

50. *The Principal of Construction.*—A liquid at rest has its surface level, for it is perpendicular to the resultant of all the forces acting upon it. Any other surface parallel to the surface of the liquid must also be level or horizontal. The spirit level is so constructed that when the bubble is at its highest point the above condition is fulfilled. Since the tube is slightly curved in the arc of a circle when the level is placed upon a horizontal surface the condition of *horizontality* is that the bubble shall *occupy* the *same*

position when the level is reversed, for if the level (Fig. 27) is the arc of a circle and is placed on LL a horizontal plane, the arcs ab and cd must be equal when the bubble or its centre is at the highest point; when the instrument is reversed the arcs will still be the same, but if the level is placed upon an inclined plane (Fig. 28) the middle point is not the highest point and reversal will cause the bubble to occupy a corresponding position at the other end.

51. The *adjustments* then are very simple; for if the level is placed upon a plane slightly inclined (Fig. 28) and then reversed the level will be correct if the positions of the bubble are the same. If they are not a small adjusting screw under one end will make them so, or the level may be placed upon a plane and readings taken of both ends of the bubble before and after reversing. The mean of the direct and reversed readings will give the position of the bubble for correctness.

52. *How to Use the Level.*—Lay it on the surface to be tested and if the bubble remains at zero in all positions we conclude that the surface is plane and horizontal.

53. *Value of a Level Scale Division in Angular Measure.*—The value of a scale division in angular measure may be determined by using a level tester, a wire of known diameter, a theodolite or transit and a kathetometer.

54. *Level Tester.*—Fig. 29 represents a level tester. Its construction is obvious from the figure. The graduated level is placed upon two Y's and the plane BH is gradually raised by the micrometer screw, the divisions of which are known. The distance from the hinge h to the screw is also known. The base AH is supplied with leveling screws so that the bubble may be placed in its middle position.

$$1 : \tan. BHA :: AH : AB.$$

$$\tan. BHA = \frac{AB}{AH}$$

$$BHA = 10L \ (e.g.) = H.$$

2. Instead of M use wire of known diameter.

3. Theodolite or transit with vertical divided scale; the method is obvious.

4. Instead of trusting to the reading of (3) make the horizontal cross-wire of the telescope coincide with the standard scale vertically placed at a known distance and observe the difference in reading for each division of the level. The same observation may be made with the cathetometer, since the level and the telescope move together.

55. *Show How to Determine the Inclination from the Divisions when ρ of the Tube is Given.*

It must be remembered that in order that a *plane surface may be horizontal* two lines in it must be horizontal. Three leveling screws are placed in a board so as to form an isosceles triangle. By placing the level parallel to a line joining two of the screws and raising or lowering one until the bubble takes its middle position and then placing the level in a line perpendicular to its first position and raising or lowering the third screw until the bubble takes its highest position the plane will be leveled.

When the bubble deviates from its middle position, a tangent to the middle of the tube deviates from horizontality an angle $=$ $206264.8'' \times \dfrac{\textit{deviation of bubble}}{\rho}$. Hence, the longer ρ the more sensitive the instrument.

In Fig. 30 let A be the position of the bubble at its middle point. AT_1 is tangent to that point. Let the bubble take the position B; the tangent is now BT_2, and the deviation of the tangent is T_1BT_2 or θ. Let ρ be the radius of curvature of the level and AB the deviation of the bubble: it is evident that ACB equals θ and that

$$1 : \theta :: \rho : AB.$$

$$\theta = \frac{AB}{\rho}$$

The constant depends on the units selected.

III. The Reading Microscope with Micrometer Attachment.

56. It is obvious that if the turns and fractional parts of a turn of the micrometer screw (Fig. 31) have been determined by com-

parison with a graduated circle that the instrument may be used for measuring small arcs.

Suppose, for example, that the head of the screw is divided into 60 divisions, and one revolution corresponds to $1'$, then one revolution equals 60 divisions $= 1' = 60''$ and one division equals $\frac{1}{60}$ of $60''$ or $1''$.

How to Use the Instrument.—Turn the graduated circle under the fixed micrometer and read the nearest coinciding lines. Turn the head of the screw until the additional distance necessary is traversed by the moving spider line and add the angular intersections as determined by the angular measure of the turns and fractional turns of the micrometer head.

IV. The Filar Micrometer.

57. The filar micrometer (Fig. 31) differs from the reading microscope in having a fixed and movable spider line.

Prob.—Measure the angular distance between two stars or other distant objects.

The instrument is used by placing the fixed line upon one object and making the movable line coincide with it. The movable line is then moved to coincide with the other object, after which the reading is taken from the head of the instrument.

Prob.—To convert micrometer readings into angular measure

Let f = focal length of the object glass.
Let p = pitch of the micrometer screw.
Let θ = angular value of one turn.

Then if θ is very small, $\tan. \theta = \dfrac{p}{f}$

5. The Optical Method.

58. *a.* **Cornu's Optical Lever.**—The Optical Lever (Fig. 32) consists of a lever mounted upon four feet, one at each end and two in the middle on a line at right angles to a line joining the two end ones. Upon the middle of the lever is a mirror which is free to rotate about a horizontal axis, this axis being parallel to a line joining the central feet. One of the end feet is adjustable so

that this end of the lever may be raised or lowered. If the lever is placed upon a perfectly plane piece of glass when properly adjusted there will be no rattling when moved.

A telescope with cross-wires and a millimeter scale placed vertically in the plane of the cross wires are also necessary parts of the apparatus.

In *using the instrument* the line joining the end feet of the lever is placed so as to coincide with the axis of the telescope placed at a distance of several meters from the lever. The lever having been placed upon the plane of glass the mirror is slightly rotated until the image of a division upon the scale is seen upon the cross-wires when viewed through the telescope which has been previously focussed for this purpose. The instrument is now ready to be used.

To *measure the thickness* of a microscope cover-glass, for example, place the glass under the middle feet. The lever will by this operation be thrown out of adjustment with one leg of the instrument raised; read the division on the cross-wire; read again after the other leg has been tilted up. The thickness of the cover-glass will then be given by the equation

$t = \dfrac{ln}{4s}$ in which t = the required thickness, l the half length of the lever and s the distance of the scale from the mirror.

59. *Proof of the Preceding Equation.*—In Figs. 33, 33*a*, MM represents the true plane surface of glass, ML_1 represents the position of the optical lever after the cover-glass of thickness t has been placed under the middle feet, ML_2 the reversed position. T is the telescope with its axis in the true plane.

Let θ be the angle the lever makes with the true plane,

l the half length of the lever,

s the distance of the scale from the mirror,

S_1 S_2 the scale with its length perpendicular to the telescope in the plane of the cross-wires. The lever in passing from the position ML_1 to ML_2 rotates through an angle 2θ. The mirror obviously rotates through the same angle 2θ. But by the principles of reflection in optics the reflected beam rotates through 4θ.

Hence, $S_1OT - S_2OT = 4\theta$.

Call S_1OT, θ_1, and S_2OT, θ_2 then

$$\tan. 4\theta = \tan. (\theta_1 - \theta_2) = \frac{\tan. \theta_1 - \tan. \theta_2}{1 + \tan. \theta_1 \tan. \theta_2} = \frac{\frac{S_1T - S_2T}{s}}{1 + \frac{S_1T \times S_2T}{s^2}}$$

But $S_1T \times S_2T$ is very small compared with s^2.

$\tan. 4\theta = \dfrac{S_1T - S_2T}{s} = \dfrac{n}{s}$ in which n is the difference between the two readings of the scale. But the angles are so small that we may take

$$\theta = \sin \theta = \tan. \theta$$

$$\therefore \theta = \frac{n}{4s}$$

Therefore $t = l \sin \theta \dfrac{ln}{4s}$

60. *Discussion.*—1. If s is very great, with nl constant, t may be very small; 2. If l is made very small, t may be very small; 3. The same result may be obtained by increasing the magnifying power of the telescope.

61. *b.* **Poggendorff's Method.**—1. *Telescope and Scale.*—If a mirror is placed parallel to a scale, when the mirror is rotated through a given angle θ, the light reflected from the mirror or the image of the scale in the mirror will be deflected through 2θ. If a telescope be placed so that the image of the scale may be viewed through it a very small deflection may be observed. This method is called the German or subjective method. In Fig. 34 let M_1M_1 represent the mirror attached to a small magnet, for example, the deflection of which is to be determined; let SS be a millimeter scale placed parallel to the plane of M_1M_1. Let the middle point R_1 of the scale be opposite the center A_1 of the mirror. Then if M_1M_1 takes the position M_2M_2 making an angle θ with its original position the light instead of being reflected back along the incident ray will be reflected to P_1 so that $P_1A_1R_1 = 2\theta$.

Then in the triangle $P_1A_1R_1$

$$1 : A_1R_1 :: \tan.\ 2\theta : P_1R_1$$

$$\tan.\ 2\theta = \frac{P_1R_1}{A_1R_1}$$

That is *the tangent of twice the angle of rotation of the mirror equals the number of scale divisions traversed by the reflected beam divided by the distance of the scale from the mirror*

$$\text{or } \tan.\ 2\theta = \frac{n}{s}$$

Cor.—If the scale is divided into millimeters and s is made a meter

$$\tan.\ 2\theta = \frac{n}{1000}$$

Hence, for great accuracy the ordinary trigonometrical tables must be used.

62. 1. If the angle of deflection is very small, the angle may be taken equal to the tangent with very little error. Suppose, for example, that πs gives 180°

then $180° : 2\theta :: \pi s : R_1T = R_1P_1$.

$$\text{Then } 2\theta = \frac{180°\ R_1T}{\pi s} = \frac{180°\ R_1P_1}{\pi s} = \frac{n \cdot 180°}{\pi s}$$

$$\theta = \frac{n \cdot 103132}{s} \text{ (nearly).}$$

Now suppose that the scale divisions are each 1 mm., and $s =$ 1000 m and $n = 100$.

$$\theta = \frac{100 \cdot 103132}{1000} = 2°\ 51'\ 53.2''$$

$$\text{while from the formula } \tan.\ 2\theta = \frac{n}{s} = \frac{100}{1000} = \frac{1}{10}$$

$$\theta = 2^\circ\ 51'\ 19''.$$

63. 2. In the comparison of deflections very little error will be committed by regarding the angles proportional to the scale readings.

For example, suppose the scale readings are 100 and 200.

By the formula $\tan. 2\theta = \dfrac{n}{s}$

$$\theta_1 = 2^\circ\ 51'\ 19.1''$$

$$\theta_2 = 5^\circ\ 39'\ 17.8''$$

The ratio between these is 1 : 1.9805 instead of 1 : 2. This reduction can be avoided by making the scale circular with the mirror at the center; this is done in Sir Wm. Thomson's Quadrant Electrometer.

64. 3. For deflections not exceeding 6° we may take

$$2\theta = \frac{17189}{s}\, n \left(1 - \tfrac{1}{3}\ \frac{n_2}{s_2}\right)$$

Sometimes a trigonometrical function is required.

$$\tan.\ ^2\theta = \frac{n}{2s}\left(1 - \left(\frac{n}{2s}\right)^2\right) = \frac{n}{2s}\left(1 - \tfrac{1}{4}\ \frac{n^2}{s^2}\right)$$

$$\sin 2\theta = \frac{n}{2s}\left(1 - \frac{3}{2}\left(\frac{n}{2s}\right)^2\right) = \frac{n}{2s}\left(1 - \tfrac{3}{8}\,\frac{n^2}{s^2}\right)$$

$$\sin \theta = \frac{n}{4s}\left(1 - \frac{11}{2}\left(\frac{n}{4s}\right)^2\right) = \frac{n}{4s}\left(1 - \frac{11}{32}\,\frac{n^2}{s^2}\right)$$

Hence, to reduce a scale reading n to corresponding arc, tangent, sine and $\dfrac{sine}{2}$, subtract $\tfrac{1}{3}$, $\tfrac{1}{4}$, $\tfrac{3}{8}$, $\tfrac{11}{32}\ \dfrac{n^2}{\underline{|r^2}}$ from n.

For considerable deflections, from geometrical considerations

$$a = \tfrac{1}{2}\tan.^{-1} \frac{n}{\underline{|L}}$$

65. 3. **The English or Objective Method** consists in removing the telescope and substituting in its place a slot across which a fine wire is stretched. The wire is illuminated by means of a lamp placed behind it. A small lens is placed in front of the wire so that an image is formed of the rays reflected from the mirror. The wire support is so tilted that the light reflected from the mirror is reflected to a scale placed either above or below the wire.

In Fig. 5 the parts of the apparatus are shown. In the Quadrant Electrometer the lens is dispensed with and a concave mirror is used to form the image upon the scale.

The scale is bent in the circular form having the centre of the mirror as the centre of curvature.

66. **6. The Goniometer** (Fig. 36) invented by Wollaston is used for measuring the angles of crystals.

67. **7. Ordinary Protractors** made of brass and horn may be used for measuring angles where great accuracy is not required.

To divide a circle originally, see article in the *Scientific American Supplement*, 1877, p. 1275.

C. AREAS.

68. **First Method.**—*By Calculation.*—The linear dimensions may be accurately measured by any of the instruments described for measuring length and the area calculated from these dimensions. The following are illustrations.

The area of a circle $= \pi r_2$.

The area of a triangle $= \frac{1}{2} b \times h$;

or let a, b, c be the lengths of the side of a triangle then

$$\frac{a+b+c}{2} = s, \text{ and the area } = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$$

or log area $= \frac{1}{2}\,[\log s + \log(s-a) + \log(s-b) + \log(s-c)]$

The area of a parabola $= \frac{2}{3} b h$.

The area of an ellipse $= \frac{1}{4} \pi a b$.

The surface of a sphere $= 4\pi r_2$.

The surface of a right cylinder $= 2\pi rl + 2\pi r^2$.

The surface of a right cone $= \pi r^2 + 2\pi r \times \frac{1}{2}$ slant height.

Sometimes the figure whose area is desired may be cut up into squares, triangles, circles or parts of circles and the areas of the component figures calculated and added together.

69. **Second Method.**—Paper divided into square millimeters or other convenient units may be used, the boundary of the area being placed on it and the number of little squares counted.

70. **Third Method.**—*By weighing.*—Copy upon a sheet of paper or metal of uniform thickness the given area. Then weigh the paper or metal. If a square of the paper or metal is also weighed the ratio of these weights will equal the ratio of the areas.

71. **Fourth Method.**—*The Planimeter.*

72. *Instructions for Using Amsler's Planimeter Accompanying Planimeter No. 6 of the Laboratory.*

73. *Needle Point Outside the Diagram.*—Put the instrument (Fig. 37) on the drawing surface with the tracing point *F* at a mark on the curve, the area of which is to be measured, press the needle point *E* slightly into the paper outside the curve, and read off the roller *D* and the counting disc *G*, taking the whole circumference of the recording roller as unit of reading (the roller need not be set to zero). The disc and roller in the figure, for example, would read 1.473, this figure being obtained thus : 1 from disc, 47 from roller, 3 from vernier.

Then move the pointer *F* round the area in the direction of the movement of the hands of a watch. The pointer having reached again the starting point, take another reading, say 3.387, and subtract the first reading from the second reading. The difference multiplied by 10 will then be the area of the curve in square inches.

Example.—Second reading 3.387
First reading 1.473

$1.914 \times 10 = 19.14$ sq. ins.

If for another example, the reading before starting the pointer had been 9.521 then the reading after circumscribing the same curve as before would be 1.435. As the travel of the roller in both cases is the same, it is evident that the zero mark of the counting disc has passed the fixed index mark and that consequently the second reading has now to be supplied with one more digit on the left before subtracting the first reading.

Thus, complete second reading 11.435
first reading 9.521

$1.914 \times 10 = 19.14$ sq. ins.

When measuring very large areas, it may happen that the counting disc makes several revolutions. If in the above example the zero mark of the disc had passed the index twice in the same direction then the complete second reading would have been 21.435 instead of 11.435.

74. *Needle Point Inside the Diagram.*—Circumscribe the diagram with the pointer in the direction of the movement of the hands of a watch noting at the same time the counting disc in order to see whether the total rotation of the recorder is a forward motion or a back motion.

This preliminary rough operation completed, proceed as before explained, following the curve *carefully* with the pointer. If the total rotation of the roller has been a *forward* motion, subtract the first reading from the second reading and *add* the difference to the figure engraved on the top of the small weight used for keeping the needle point in its place. The sum multiplied by 10 will then be the area of the curve in sq. ins.

Example.—Second Reading $4.537 - $ First Reading $3.438 = 1.099$; $1.099 + 27.132$ (which is the figure on the weight) $= 28.231$; $28.231 \times 10 = 282.31$ sq. ins. (The figure 27.132 is slightly different for different instruments).

If the total rotation of the roller is a *back* motion, subtract the second reading from the first reading and *subtract* the difference from the figure on the weight.

Example.—Complete first reading 12.732
second reading 8.235

4.497

$$\begin{array}{r} \text{Figure on weight } 27.132 \\ 4.497 \\ \hline 22.635 \end{array}$$

$$22.635 \times 10 = 226.35 \text{ sq. ins.}$$

Note.—When the mark *O* of the roller is at the mark *O* of the vernier a mark on the counting disc should be opposite the fixed index mark. Any slight non-coincidence due to play between roller and disc may readily be allowed for in taking readings.

The area corresponding to a total revolution of the roller — 10 square inches in the above example is engraved on the weight with the constant before mentioned.

75. *Where the sliding tube is used* adjust the sliding tube on the bar so that the index mark on the tube coincides with one of the marks on the bar. The unit of area is engraved to the right of the corresponding mark. Then proceed in exactly the same way as before explained.

Example.—Area required in square feet. Slide the tube on the bar so that the index of the former coincides with the mark denoted by 0.1 square foot. Suppose the dimensions of the diagram allow the needle-point to be placed *outside*.

$$\begin{array}{rl} \text{Then second reading} & 8.311 \\ \text{first reading} & 2.322 \\ \hline & 5.989 \times 0.1 = 0.5989 \text{ sq. ft.} \end{array}$$

If, on the other hand, the needle had been placed *inside* the curve and if it had been found by a previous rough test that the total rotation of the roller was a forward motion, then

$$\begin{array}{lr} \text{second reading} & 5.423 \\ \text{first reading} & 3.004 \\ \hline & 2.419 \end{array}$$

$$\begin{array}{r} 2.419 \\ \text{Figure on top of bar just over the mark } 20.741 \\ \hline 2.3160 \times 0.1 = 2.316 \text{ sq.ft.} \end{array}$$

The figures on the top of the bar are slightly different for different instruments.

76. *To Determine the Mean Height of Indicator Diagram with Planimeter No. 6, Laboratory* (Fig. 39).—For doing this take by shifting the tube on the bar the diagram lengthwise between the steel points on the upper side of the instrument as shown in the figure. Then place the planimeter without altering the relative position of tube and bar in the usual way upon the drawing board the needle point outside the diagram and circumscribe the diagram with the pointer. The difference of the readings at the beginning and at the end of the operation divided by 0.4 is then the mean height in inches.

If the diagrams for up and down stroke are measured jointly divide by 0.8 instead of 0.4.

Mean pressure = mean height × scale of spring of indicator
Supposing the scale of spring in the above example is 1 in. = 80 lbs. per sq. in., then

$$\text{mean pressure} = \frac{0.448 \times 80}{0.4} = 89.6 \text{ lbs. per sq. in.}$$

The number of pounds per inch of height being usually a multiple of 4. The arithmetical work is thus extremely simple.

D. VOLUMES.

77. **First Method.**—*By Calculation.*—If the body is a regular figure, the linear dimensions may be measured by the instruments described for measuring length, and the volume calculated., *e.g.*,

The volume of a cylinder $= \pi r^2 l$.

The volume of a sphere $= \frac{4}{3}\pi r^3$.

The volume of a cone or pyramid = area of base × ⅓ altitude.

78. **Second Method.**—*By Weighing.*—*Principle Involved.*—Let m be the mass of a body determined by the balance, V its volume, and δ the mass of unit volume or density, then $m = V\delta$ and $V = \frac{m}{\delta}$. The specific mass may be obtained for different temperatures from carefully constructed tables.

Prob.—Find the volume of the interior of a graduate by filling with distilled water and weighing.

79. **Third Method.**—*By Using Graduated Vessels.*—This method is applicable to liquids. The vessels may be

(*a*) Measuring flasks graduated to litres, half litres, etc.

(*b*) Pipettes (Fig. 40) which are tubes open at both ends, one end being drawn to a point. They are graduated to 100, 50, 25, 10, 5 and 1 centimeters.

(*c*) Burettes (Fig. 41). Mohr's (*e.g.*) are graduated tubes having a stop-cock at the lower end so that a small quantity of the liquid may be drawn off. The amount drawn off is measured by the changing position of a glass float, Erdman's having a line engraved around its body.

The methods of calibrating these vessels may be found in books treating of laboratory measurements.

E. MASS.

80. This is *effected* by *means* of the *balance* for the theory and details of which see *Elements of Natural Philosophy*, §§133–138.

F. SPECIFIC DENSITY.

81. The specific density may be determined by various methods.

1. By the Hydrostatic Balance, §405–1.
2. By the Specific Gravity Flask, §405–2.
3. By Nicholson's Hydrometer, §405–3.
4. By Difference of Level, §391.
5. By Specific Gravity Bulbs (Fig. 42), §405–5.
6. By Hydrometers, §408.
7. By Jolly's Balance.

G. TIME.

82. 1. Pendulum.

83. 2. True time with sextant or transit.

84. 3. Chronograph.

85. 4. To measure small intervals.

a. Flow of mercury through orifice ; weight in given time.

b. Flow of clepsydra.

c. Tuning fork.

H. FORCE.

86. 1. By measuring m and a directly since $F = ma$.

1. Atwood's Machine. Daniell, p. 29.

87. 2. Direct Counterpoising.

Illustrated by measuring adhesion of glass plate to surface of mercury. Daniell, p. 31.

88. 3. Indirect Counterpoising.

Spring Balance.

Dynamometer.

89. 4. Method of Oscillation.

$$f_1 : f_2 :: v_1^2 : v_2^2 :: n_1^2 : n_2^2.$$

I. WORK OR ENERGY.

90. 1. Ergometers. $F_r s$ or Fs.

J. SPEED OF A ROTATING SHAFT.

The speed of a rotating body is generally given in turns or fractional parts of a turn per minute or second instead of in terms of angular velocity (ω).

The angular velocity equals the number of turns in a second of a point at unit's distance from the axis or $\omega = 2\pi n$. Various methods may be employed to get this value.

91. 1. **Mark on a Wheel.**—If the number of turns in a given time is small and the wheel large, a chalk or other mark may be made near the circumference and the number of times this mark comes to a fixed point in a minute may be observed. This number divided by 60 will give the number of rotations in a second.

92. 2. **Mark on Belt.**—If a belt runs over a pulley, a chalk mark or lacer may be employed to determine the number of times the marked part of the belt comes to a fixed point. If the length of the belt and the circumference of the pulley are known the speed may be found: for, one complete turn of the belt corresponds to the length of the belt and the number of turns in a minute multiplied by the length corresponds to the whole travel of the belt in a minute. If this product be divided by the circumference of the pulley the required speed is found or

$$\text{the number of revolutions of the pulley} = \frac{\text{length of belt} \times \text{by the number of times the mark comes to the fixed point}}{\text{circumference of the pulley}}$$

There must be no slip of the belt; if there is the method is inaccurate. A cord or tape may be arranged so that the slip is negligible.

93. 3. **Lead Pencil and Moving Card.**—Sometimes a lead pencil may be tied upon a shaft with its length parallel to the axis: if a card is now held in a fixed position the point of the pencil just touching it a number of superposed circles will be drawn upon it. If the card be moved in its own plane the circles will overlap and the number drawn in a minute may be counted.

94. 4. **Spool of Thread.**—In certain cases where friction must be avoided one end of a thread may be connected to the rotating part and the thread may be unrolled from the spool. The number of turns of thread on the axle may be counted and the desired quantity obtained. The thread must not be unequally stretched.

95. 5. **The Acoustic Method.**—This method has been successfully applied to small electric motors running at very high speed. Attach to the axle a Savart's wheel with three teeth and having a light steel spring touching it lightly. To a tachometer apply a similar wheel and spring. Work the tachometer by hand until the same note is given out by it as by that of the motor axle. The number of revolutions shown by the tachometer is the number required.

96. 6. **The Telephone.**—If the springs used in the preceding method be placed in the circuit of two batteries each containing a telephone equality of pitch will be noticed when the number of revolutions is the same. A telephone with double windings may be used instead of the two.

97. 7. **The Ordinary Counter** is an endless screw having an index which moves over a graduated face. The axis of the screw has a pointed end which may be introduced into the counter sink of the shaft. The number of revolutions is read off of the face.

98. 8. **The Dial Counter** consists of a combination of disks with figures upon the edges upon which the exact number of revolutions may be read. Sometimes a hand points to the number of revolutions upon a graduated dial.

99. 9. **Contact Wheel.**—A small wheel may replace the pointed end of the tachometer. The circumference of this wheel is covered with a rubber band. If the rubber circumference be applied to the face of a pulley the number of revolutions per second may be found. Now measure the radii of the pulley and of the tachometer wheel. The number of revolutions will be inversely as these radii.

K. TORQUE OR THE MOMENT OF A FORCE.

This value is important in determining the rate of work or horse-power of an engine. Various instruments are employed among which are the following:

100. 1. **The Prony Friction Brake.**—In this brake the friction of a lever against a wheel attached to the shaft is balanced by weights applied to the distant end of the lever. When the system is in equilibrium the moment of the added weights equals the

moment of the friction. Fig. 43 represents the usual form. A counterpoise is sometimes applied to the short arm to balance the weight of the lever. If this is not done the moment of the weight of the beam must be added to the moment of the weights placed in the pan.

Fig. 44 represents a better form.

101. 2. **Modified Pony Brake.**—A band of iron upon which blocks of wood are fastened may take the place of the blocks of the ordinary brake. The lever arm may be arranged to press upon a common platform scales. (Fig. 45.)

102. 3. **Belt Brake.**—Instead of the above arrangement, a leather belt to which blocks with projecting edges are fastened to prevent the belt from slipping from the pulley may be used. One end of the belt is hooked to the floor and the other end has a pan fastened to it upon which weights may be placed. (Fig. 46.)

103. 4. **Modification of 3.**—Instead of having one end of the belt fastened to the floor a spring balance may be introduced as in Fig. 47.

Any pull registered by the balance must be deducted from the weight placed in the pan. Of course the weight of the spring balance must be added to the pull at A.

Let $W =$ the weight in pounds for equilibrium when the motor is at rest.

$W_1 =$ the weight in pounds for equilibrium when the motor is in motion.

Then $(W - W_1)$ (circumference in feet $+ \frac{1}{2}$ thickness of belt on each side) (number of pulley revolutions per minute) $=$ ft. lbs. of mechanical energy.

104. 5. **Rope Brake.**—The most convenient, the simplest and for continuous action the best friction brake is made of a rope. In all the preceding forms soapy water must be applied to prevent heating. In the rope brake no danger arises from this cause. Fig. 48 represents the best form. An endless rope upon which blocks with overhanging edges are fastened to prevent slipping off is placed around the pulley. One loop of the double rope is hooked upon a spring balance suspended from the ceiling: the other loop is attached to another spring balance fastened to the

floor. The rope is made taut by a running screw which passes through the floor, or by means of a turn buckle. The difference between the readings of the upper balance and the sum of the weight of the lower balance and its reading gives the value of the resistance required or tension on the rope.

105. 6. **The Kapp Brake.**—For determining the torque of of small electric motors Kapp modified the ordinary equal arm balance as is shown in Fig. 49.

It will be observed that one end of the brake cord is attached to a point on the line joining the fulcrum with the points of suspension of the pans; while the other end is attached below this line. It is evident from this arrangement that if the balance tilts, the lever arm of A_1 is increased while that of A_2 is decreased and thus a compensating device is formed.

The resistance having been determined by any of the preceding methods its lever arm may be measured and the product of the two taken which is the torque required or $F_r S$.

L. HORSE-POWER.

The power of a machine or the *rate* of performing work is usually measured in horse-power. One horse-power is the work performed in lifting 33,000 pounds through one foot in one minute or 550 foot-pounds per second. (315.)

Hence if the speed and torque be found by the foregoing methods

$$\text{Mechanical horse-power} = \frac{F_r \cdot l \cdot 2\pi n}{33{,}000}$$

$$\text{Electrical horse-power} = \frac{\text{Volts} \times \text{amperes}}{746}$$

M. EFFICIENCY.

$$\text{Efficiency of dynamo} = \frac{\text{Electrical power}}{\text{Mechanical power}}$$

$$\text{Efficiency of motor} = \frac{\text{Mechanical power}}{\text{Electrical power}}$$

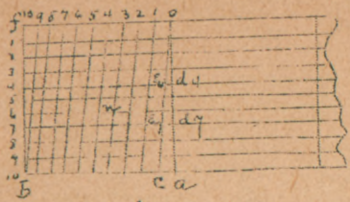


Fig. 1

Fig. 2

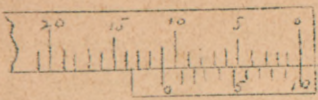
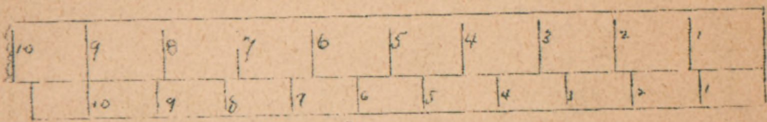


Fig. 3



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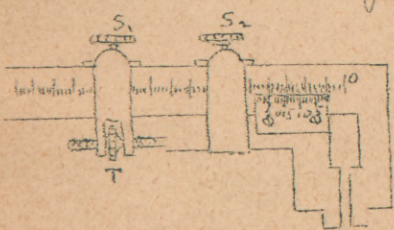

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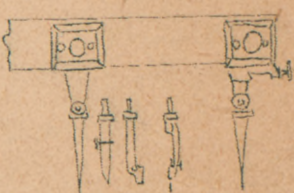

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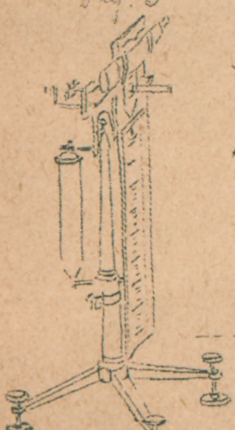


Fig. 7

Fig. 7a

Fig. 7b

Fig. 7c

Fig. 7d

Fig. 8

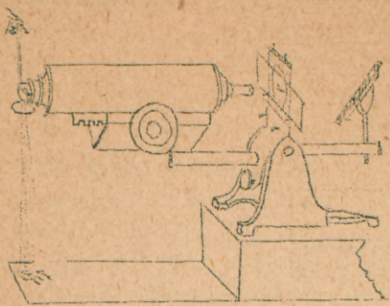


Fig 9

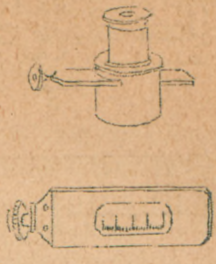


Fig. 10

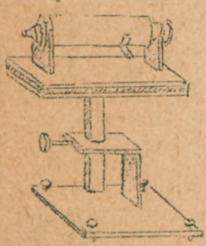
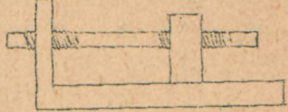


Fig. 11

Fig. 12



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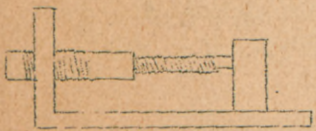

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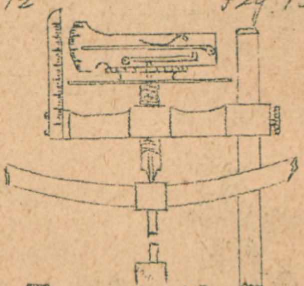

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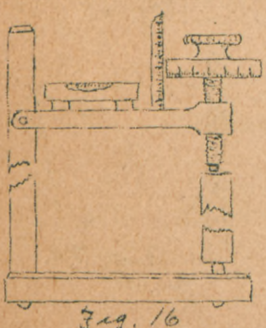

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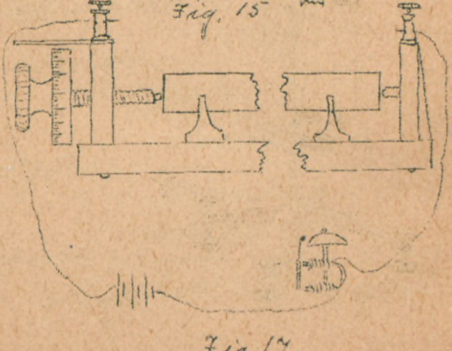

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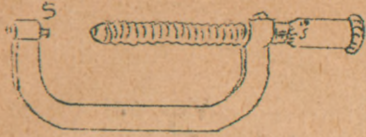

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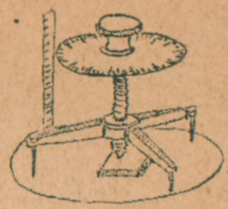

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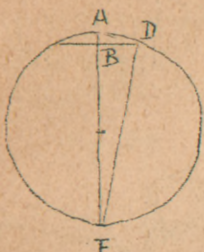

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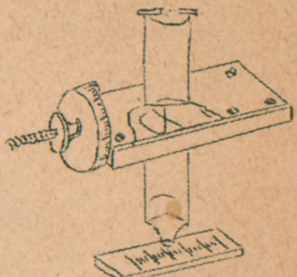

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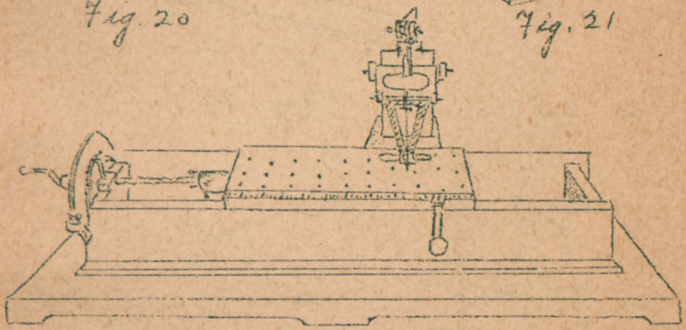

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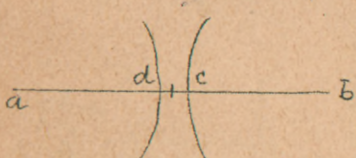


Fig. 23

Fig. 24

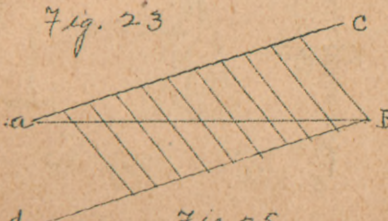


Fig. 25

Fig. 26

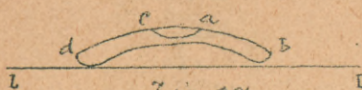
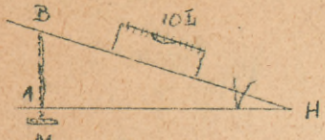


Fig. 27

Fig. 28



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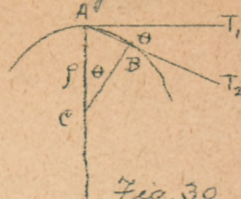
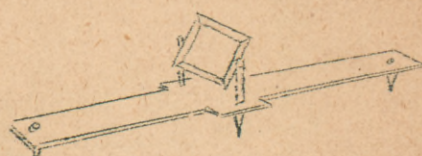


Fig. 30

Fig. 31



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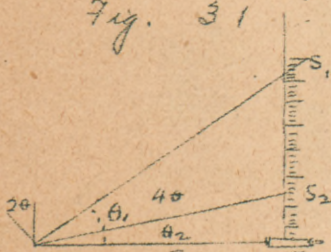
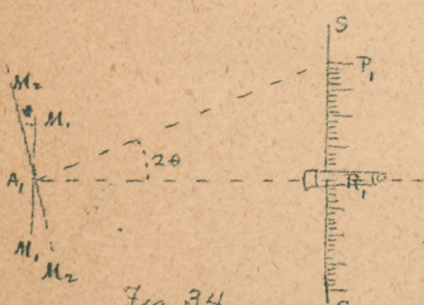


Fig. 33

Fig. 33a



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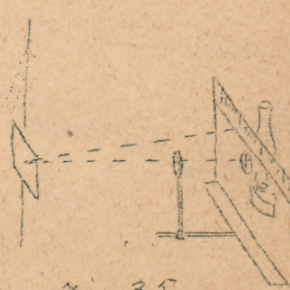

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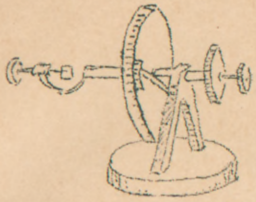

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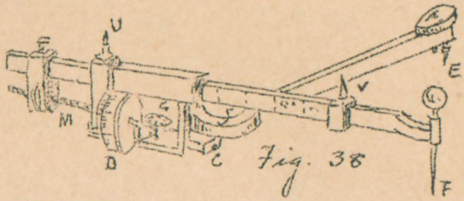

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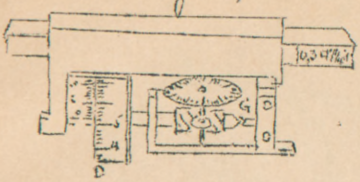

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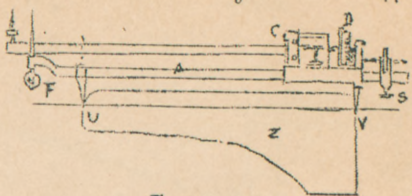
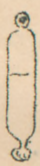


Fig. 40

Fig. 40 a.



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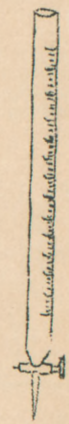

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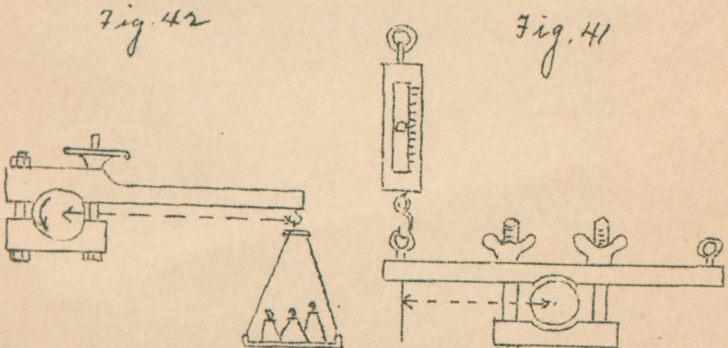

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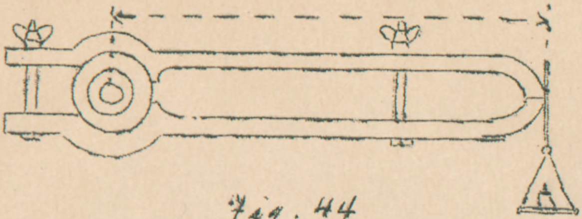

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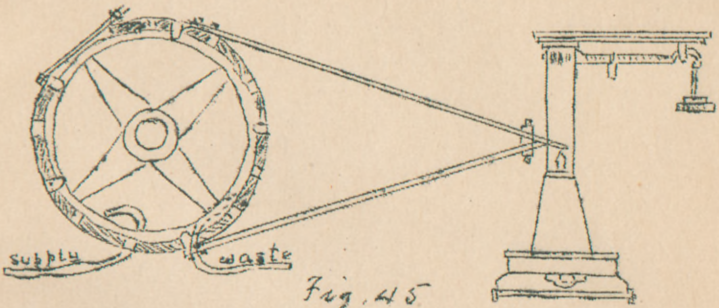

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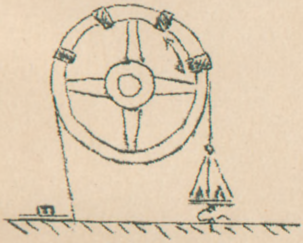

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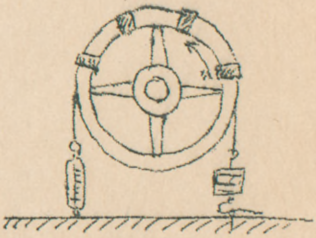

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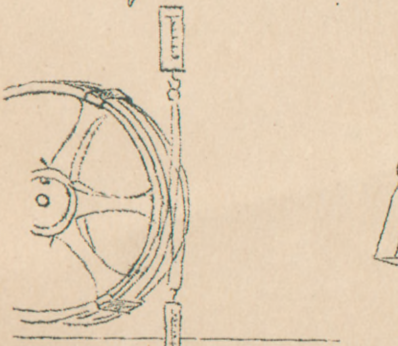

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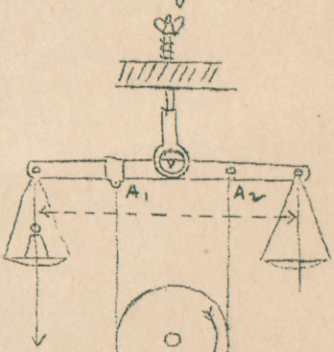

Fig. 49