Stevens (W. Le C.)

The Stereoscope × × × ×

WITH THE COMPLIMENTS OF

WALTER LE CONTE STEVENS,

40 WEST 40TH STREET,

NEW YORK.

flashing

Reiss, Graham Bell, Edison and Hughes are those deserving to be recorded.

[...]ist electricity has thus furnished us with the means of flashing our thoughts by record or by voice from place to place, its use is now gradually extending for the achievement of such quantitative effects as the production of light, the transmission of mechanical power, and the precipitation of metals. The principle involved in the magneto-electric and dynamo-electric machines, by which these effects are accomplished, may be traced to Faraday's discovery in 1831 of the induced current, but their realization to the labors of Holmes, Siemens, Pacinotti, Gramme, and others. In the electric light, gas-lighting has found a formidable competitor, which appears destined to take its place in public illumination, and in lighting large halls, works, &c., for which purposes it combines brilliancy and freedom from obnoxious products of combustion, with comparative cheapness. The electric light seems also to threaten, when subdivided in the manner recently devised by Edison, Swan, and others, to make inroads into our dwelling-houses.

By the electric transmission of power, we may hope some day to utilize at a distance such natural sources of energy as the Falls of Niagara, and to work our cranes, lifts, and machinery of every description by means of sources of power arranged at convenient centres. To these applications the brothers Siemens have more recently added the propulsion of trains by currents passing through the rails, the fusion in considerable quantities of highly refractory substances, and the use of electric centres of light in horticulture as proposed by Werner and William Siemens. By an essential improvement by Faure of the Planté Secondary Battery, the problem of storing electrical energy appears to have received a practical solution, the real importance of which is clearly proved by Sir W. Thomson's recent investigation of the subject.

It would be difficult to assign the limits to which this development of electrical energy may not be rendered serviceable for the purposes of man. * * *

[FROM THE AMERICAN JOURNAL OF SCIENCE, VOL. XXII, NOVEMBER, 1881.]

ART. XLVII.—*The Stereoscope, and Vision by Optic Divergence;* by W. LECONTE STEVENS.

DURING the last twelve years, Professor Joseph LeConte has published in this Journal a series of articles on Binocular Vision, in one of which he refers to a gentleman with normal eyes "who could combine ordinary stereoscopic pictures with the naked eyes beyond the plane of the pictures, even when the distance between the identical points was greater than the distance between the centers of his pupils." He adds, "It would be curious to inquire, at what *distance* and of what *size*, according to the laws of vision, the stereoscopic image ought to seem in this case."*

While conversing with this gentleman,† about three years ago, it was discovered that I possessed the same power; and since that time no stereograph has been found on which identical points were too far apart to secure binocular fusion with the naked eyes. Not until last spring, however, did I begin any careful investigation of these phenomena. Professor LeConte has investigated the phenomena of ocular convergence very fully, and has developed a system of diagrammatic representation far more consistent than any previously published. I have tested all the experiments on this subject that he has described; and my results have been either identical with his, or as closely approximate as could be reasonably expected. To avoid repetition of what has been already sufficiently established I shall assume that the reader is familiar with the contents of Professor LeConte's papers.‡ It will be found convenient to study optic divergence especially in connection with the stereoscope.

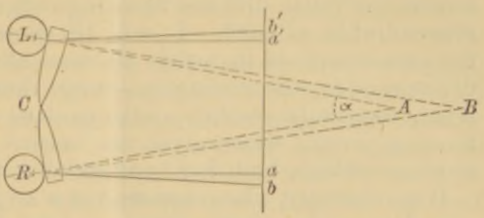
In normal binocular vision the two eyes may be regarded as human cameras occupying slightly different positions, from which are obtained simultaneous views of the point upon which the visual axes are converged. The apparent distance of this point is mainly determined by the intersection of these axes, if the optic angle is large enough to be readily appreciable. In reading ordinary print with comfort the optic angle is rarely less than 12°.

The method of preparing photographs for the stereoscope is too familiar to describe. It is usually assumed that, when these are viewed through the instrument, the lenticular prisms are so adjusted that rays are deviated into the observer's eyes from corresponding points of the stereograph, as if coming from single objects in front; so that he may easily imagine his own

* III, ix, 162–163, March, 1875. † Mr. James Wood Davidson, of New York.
‡ This Journal, II, vol. xlvii, pp. 68 and 153; III, vol. i, p. 33; vol. ii, pp. 1, 315, and 417; vol. ix, p. 159.

eyes to replace the photographer's cameras, and the convergence of his visual axes to replace that of axes from some point in the landscape upon which these cameras have been directed. In fig. 1 let *aa'* be the foreground interval and *bb'* that for the background on the stereograph; then the foreground appears at A and the background at B.

1.



To determine the apparent distance of A, let *i* stand for the observer's interocular distance, RL; *α* for the optic angle, RAL, and D for the apparent distance required. Then, if *a* and *a'* be symmetrical,

$$D=\tfrac{1}{2}i\ \cot\tfrac{1}{2}\alpha.$$

From this equation it is seen that if *α* be reduced to zero by making the axes parallel, D becomes infinite and there is no intersection. If *α* be made negative by causing the axes to pass from convergence through parallelism into divergence, D becomes negative and the intersection is behind the observer's head. In either of these cases a physiological impossibility is implied, if we accept the theory that the apparent distance of the combined external image is determined by the intersection of the observer's visual axes. If, therefore, distinct binocular vision is attainable with the axes either parallel or divergent, and any judgment of distance is possible, however faulty it may be, this fact is sufficient to prove that the theory is imperfect, and other elements must be sought for the determination of the judgment of distance in vision through the stereoscope.

In normal binocular vision axial convergence is the most important one of several elements which together determine the apparent distance of the point of sight, provided the real distance of this be near the lower limit of distinct vision. In such cases the formula just deduced is applicable with little or no modification. If *i* stand for the distance between two photographer's cameras directed to the same point in a landscape, the formula is also applicable to them, provided there be no lack of uniformity in the media through which the rays pass. In normal vision, moreover, both the focal and axial adjustments of the eyes are consensually adapted to the distance of the object regarded, and the deliverances of the muscular sense from the ciliary and rectus muscles conduce to the same judgment of distance. This judgment is the product of the past experience of the individual, and its accuracy must depend largely upon his acquired skill in interpreting muscular sensations, comparing external relations, and remembering the results of such comparisons. If by any means the axial adjustment can be made to differ considerably from that which usually accompanies a given focal adjustment, binocular vision is to that extent abnormal, and the resulting judgment of distance is correspondingly vitiated. It will be shown that vision through the stereoscope is in nearly all cases abnormal, and that optic divergence is not uncommon among those who use this instrument, especially among young persons whose interocular distance is small, whose eyes are normal, and whose power of accommodation, both focal and axial, is hence large

If an observer, who possesses but a single eye, looks out upon a landscape, the relative distance of the different objects viewed may be roughly estimated in terms of some standard arbitrarily chosen, so long as they are not precisely aligned with his eye. The judgment is less accurate as the angular separation of the objects becomes less, and as there are fewer of them at moderate distances with which to compare the rest. Always, and often unconsciously, he employs one or more of the following elements in judging the distance and form of each object regarded.

I. Near objects subtend larger visual angles than remote objects of equal size.

II. Near objects are seen more distinctly than those that are remote. The illusion of distance may hence be produced by decreasing the brightness of the object viewed, by changing the nature of the medium, or by increasing the contrast between light and shade.

III. Near objects, that are almost aligned with those which are remote, partly cover them. Covering objects are judged nearer than those covered.

IV. Familiarity with the dimensions of known objects when near enables us to compare them when remote and thereby judge their relative distance.

V. By moving from one standpoint to another and comparing the new view with what is retained in memory of the previous one, parallax of motion thus contributes to the formation of a judgment of both distance and form.

The mere synopsis of these elements is all that is necessary; separately they are familiar enough, and to illustrate them would be easy. Every one of them may be employed in the use of each eye, either separately or in conjunction with its companion. For distances of more than 240^m the binocular observer has no advantage except that two eyes receive more light than one, and the combined external image hence appears brighter and more distinct. All of them except the last may be imitated in pictures, and some of them, notably the second, may be heightened by the magnifying effect of lenses. In study-ing binocular vision they must be eliminated as far as possible; and all except the first may be nearly eliminated by using only skeleton pictures. In ordinary stereographs their combined effect is usually greater than that due to binocular perspective.

If for convenience we apply the term physical perspective to the combined effect of the elements enumerated, then that of focal and of axial adjustment may be called physiological perspective. The latter might be regarded as mathematical if the theory set forth at the beginning of this paper were strictly applicable in all cases. It is well known that focal adjustment does not vary sensibly for distances of more than 6^m, and that its effect is greatest just beyond the near limit of distinct vision, which is also about the average distance at which a stereoscope card is held when regarded. It is also well known that in normal binocular vision, the convergence of axes does not vary sensibly for distances of more than 240^m. In abnormal vision convergence may be diminished until the limit of parallelism is passed; and the judgment of distance continues to be affected by the relaxation of the interior rectus muscles, or contraction of the exterior rectus, or by both, while the focal adjustment is still adapted to the distance of the object in front held as near as convenient. The judgment of distance which results from the conflict of elements produced by this unusual coördination of muscular actions is necessarily by no means mathematical in accuracy.

While the possibility of securing divergence of axes for normal eyes has been long known, no analysis of the visual phenomena in binocular vision by this method has appeared in print, so far as I am aware. Professor LeConte's diagrams show how to determine the apparent direction of the object viewed, but he says,* "there is no point of sight." There is certainly none determined by intersection of visual axes. In reference to images perceived by abnormal vision, Helmholtz says,† "we judge them according to their nearest resemblance; and in forming this judgment we more easily neglect the parts of the sensation which are imperfect than those which are perfectly apprehended." In combining stereoscope pictures by axial divergence, either with or without the instrument, I secure vision so clear that no defect is appreciable at any point however carefully scrutinized; it does not seem necessary then to assume that any parts of the sensation are neglected. The case was very slightly otherwise during my first experiments in divergence. He makes also the following observation, that I translate from the French edition, which is the latest, of his work on Physiological Optics:‡ "When we compare a stereoscopic

* This Journal, III, vol. ix, p. 163.

† Popular Lectures on Scientific Subjects, 1st series, p. 307.

‡ Optic Physiologique, p. 828, edition 1867.

image, observed by divergence of the visual lines, with very remote real objects visible above the stereoscope, such as a remote chain of mountains, the stereoscopic image appears to us much more remote than real objects the most distant." The apparent anomaly of binocular vision without convergence of axes he refers, in this connection, to our "comparing the sensation produced with that which resembles it the most, and which is not distinguishable from it but by feebler convergence, that is, with what very remote objects give us." So far as axial divergence alone is effective, I am unable to sustain Helmholtz's observation; nor is it sustained by those whom I have tested, every one of them giving results closely accordant with my own, care having been taken to prevent any previous knowledge of my object in questioning them. All that is essential is to secure axial divergence and compare the binocular effect with the monocular effect of the same picture, if the original landscape be not present. Before me is a stereograph representing Alpine scenery, which I combine binocularly, with from 2° 17′ to 2° 40′ of divergence, as foreground and background are successively regarded. On closing the left eye, the apparent distance of a remote mountain is not perceptibly diminished; indeed on account of the decreased brightness of the monocular image, the mountain seems slightly farther. To eliminate physical perspective as much as possible, this being always strong in pictures of landscapes, a stereograph is now taken, representing a white marble statue against a dark background; the stereographic interval can be varied at will, the card having been cut in two.. Placing this in the stereoscope, the two pictures are drawn apart until 5° of axial divergence is attained, the experiment being made at a window from which an extensive landscape can be seen for the purpose of comparison. By no effort of imagination can I estimate the apparent distance of the statue to be more than 10^m. A stereograph representing a skeleton cone is now substituted, but with the same result.

It may be safe to say therefore that if Helmholtz was examining, by axial divergence in the stereoscope, a picture of the same landscape that lay actually before him, the mountains in the picture appeared farther off than those with which they were at once compared by normal vision with both eyes, all the elements of physical perspective being the same in both cases. This is probably what he meant. But his remark is not necessarily or generally applicable when stereograph and landscape are unrelated. Mere divergence of axes is not enough to reverse physical perspective, but may modify it to some extent and introduce special illusions.

[To be continued.]

[From the American Journal of Science, Vol. XXII, December, 1881.]

The Stereoscope, and Vision by Optic Divergence; by W. LeConte Stevens.

In a previous article* it has been shown that Brewster's theory of binocular perspective is insufficient to explain vision through the stereoscope when the visual axes diverge. It takes account of only one of several elements which combine to determine the judgment of distance, and the significance of this

* This Journal, Nov., 1881.

should be referred to the sensation of muscular strain rather than to the intersection of visual lines.

The effect of varying the tension of the rectus muscles of the eyes in modifying the estimate of relative distance has been applied in Wheatstone's pseudoscope* and in his reflecting stereoscope, though no reference in this connection has been distinctly made to anything beyond variation of convergence. The following experiment is not difficult. Upon a large sheet of paper a series of vertical parallel lines are drawn, 50^{mm} apart; the last line of this series forms the first of a second series 60^{mm} apart, and in like manner this introduces a third series 70^{mm} apart. Gazing at the first series, as if regarding a remote object, the paper being 1^{m} distant, the images of the lines are soon combined by diminished convergence. Passing slowly to the second series, the convergence is still farther diminished, and it passes into divergence when the third is successfully combined. The apparent distance of the first series I estimate at $2^{m}\cdot5$, of the second about 3^{m} and of the third about $3^{m}\cdot5$. By intersection of axes, the first should be 6^{m}, the second infinity, and the third -6^{m}, my interocular distance being 60^{mm}. The experiment may be varied in many ways; different observers form different estimates of distance, but I have found none who succeeded in attaining divergence thus without observing an apparent recession of the external image.

To ascertain whether divergence of axes is unconsciously practiced in the use of the stereoscope, I examined 166 stereographs taken at random and found the foreground interval to vary between 60^{mm} and 95^{mm}, the mean being $72^{mm}\cdot9$. The average interocular distance for adults is a little less than 64^{mm}; to combine without the stereoscope, therefore, divergence is nearly always necessary. To ascertain the mean deviating power of the lenticular prisms used in the best instruments, 30 pairs were obtained through the courtesy of Mr. H. T. Anthony, of New York. With but slight variation, the focal length was found to be $18^{cm}\cdot3$. Mounting each pair in succession, parallel rays 64^{mm} apart were transmitted and received upon a screen $18^{cm}\cdot3$ distant. The mean interval between points of light caught on the screen was $79\cdot1^{mm}$; hence if rays be sent from corresponding stereograph points, separated by a wider interval than this, they will be not quite parallel after emergence from the prisms, and the eyes must diverge to receive them; 80^{mm} may be taken as a limit beyond which most persons will find divergence necessary if binocular combination in the stereoscope is successfully attained. As this limit is not unfrequently exceeded, axial divergence, unconsciously attained, is quite common, though

* Phil. Transactions, 1852, part I; or, Phil. Magazine, 1852, pp. 506–523.

in extent it rarely exceeds 2° or 3°. I have attained 7°, and Helmholtz * gives 8° as his limit. Several persons of my own acquaintance have been found able to secure divergence without the stereoscope, and their estimates of the apparent distance, size and motion of the external image under various conditions have not differed much from my own.

In the discussion of normal binocular vision, the expression "point of sight" may be applied theoretically to the intersection of optic axes. Its *apparent* position, though not mathematically determined, may be estimated with more or less error, according to the skill of the observer. But in discussing the stereoscope such a definition has to be totally abandoned. The point of sight then is only the point in space to which the observer mentally refers the binocular combination of images formed on corresponding retinal points, where the visual axes, whether convergent, parallel or divergent, meet the retinas. Its apparent position has to be estimated, not determined by intersection of lines. In this estimation the relation between the visual axes is only one of a number of elements that are combined in the formation of a judgment, whether vision be normal or abnormal. Even if stereographs are selected from which physical perspective is in great measure eliminated, the optic angle may be negative; and, when positive, its effect is still antagonized by the disturbance of coördination between focal and axial adjustments, or by the observer's unconscious recognition of the circumstances under which he has been accustomed to view an object of the kind represented. A mountain will never be judged to be so near as a mere diagram, even though the axial relations be similar in viewing the pictures separately. In the stereoscope before me I place a pair of conjugate diagrams representing a skeleton cone, alternately approximating and separating them, in a transverse vertical plane, so that the optic angle varies between +8° and −3° 45′. The apparent distance varies between 30^{cm} and 40^{cm}; if determined by the optic angle it should vary between $+43^{cm}$ and -92^{cm}. The distance of the card remains constant, and tends to keep the focal adjustment so, while the eyeballs are rotating outward, tending to produce adaptation of focal adjustment to a greater distance, the two adjustments being usually consensual. We are in the habit of associating diminution of convergence with increase of distance of the object of sight. As long as the eyeballs continue rotating outward, therefore, the object appears to recede and to enlarge correspondingly, the recession being fastest during the change from marked convergence to parallelism. It does not seem possible to express this apparent rate in mathematical terms.

* Optique Physiologique, p. 616.

The experiment just described does not imply any unusual conditions in the stereoscope except that the higher value, 8°, given to the optic angle is greater than usual. Assuming $72{\cdot}9^{mm}$, the mean already found for the stereographic foreground interval, the corresponding angle of convergence after allowing for deviation of rays is a little less than 2°; the intersection of axes is hence still far from the point to which the focal adjustment is adapted. This fact explains the difficulty experienced by so many persons in obtaining distinct vision through the stereoscope, especially those who have passed beyond middle age and lost in great measure the power of focal accommodation.

Most of the stereographs in common use are pictures in which physical perspective is strong. When these are properly mounted and viewed in the stereoscope the chief advantage gained by use of this instrument seems to be that it necessitates variation in the relation between the ~~optic~~ visual axes, in order that perfect binocular combination of the different parts of the superposed retinal images be secured in the subjective Cyclopean,* or combined binocular, eye. If there is perfect superposition of retinal points on which the foreground of the stereograph is imaged, there is necessarily imperfect superposition of those on which the background is imaged. If the attention is then given to the background, slight outward rotation of the eyeballs is necessitated, and this is habitually associated with recession of the point of sight. Whether with axial divergence binocular relief is instantly perceptible, as in Dove's experiments with axial convergence, by illumination of the stereograph with the electric spark, I am unable yet to say; I hope to test this at no distant day. It should be so according to Professor LeConte's theory of binocular perspective.†

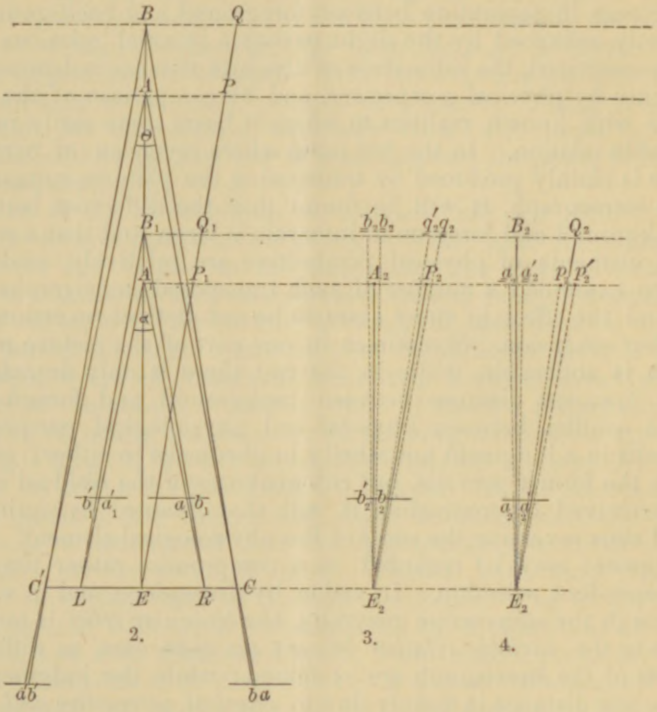
What has been generally given and accepted as the mathematical theory of the stereoscope applies strictly, but only to the relations involved in taking the photographs with cameras appropriately placed, so that the axes of the lenses converge upon some point of the object pictured. When the negatives are once fixed and proofs from them so mounted that corresponding points from the pair are focalized upon corresponding retinal points for the observer who binocularly combines them, with or without the stereoscope, the relation between the different parts of the fields of view combined undergoes no sensible variation, real or apparent, except between the limits fixed by difference between the stereographic intervals in the background and foreground respectively. If the eyes are comfortable, after binocular combination is attained, it makes little

* This Journal, III, vol. i, p. 33 et seq.
† This Journal, III, vol. ii, p. 3.

difference whether, at a given moment, the visual axes are convergent, parallel, or divergent. The combined external image as a whole is made to appear nearer by convergence and farther by divergence, but this has no perceptible effect upon the ratio between the distances of its different parts. Though the distinctness in separation between foreground and background is greatly enhanced by the slight variation in axial relations that is necessitated, the estimation of absolute distance is determined mainly by physical perspective, and by comparison of the picture with known realities to which it bears some easily recognizable relation. In the few cases where reversion of perspective is plainly produced by transposing the pictures composing the stereograph, it will be found that the difference between background and foreground intervals is large, and that some of the elements of physical perspective are relatively weak. I have examined a number of such transposed stereographs and found the effect in many cases to be not distinct reversion but rather confusion. Sometimes in one part of the picture reversion is noticeable, while in the rest there is only decrease in the apparent distance between background and foreground. The conflict between physical and physiological perspective results in a judgment not wholly in obedience to either; generally the former prevails, but the weakness of the residual effect is perceived by contrasting it with that obtained by squinting and thus reversing the sense of the physiological element. The judgment may be regarded as a compromise rather than an independent selection. In vision by divergence, and in vision through the stereoscope generally, the *binocular relief* is largely due to the *variable relation between the optic axes*, as different parts of the stereograph are examined; while the judgment of absolute distance is mainly due to *physical perspective and comparison with remembered realities*; it is modified by focal adjustment, and is in practice nearly, but not quite, *independent of the optic angle.* This remark would not apply if the optic angle were very large. visual

No diagrams can ever represent with perfect accuracy the apparent positions of objects seen in the stereoscope. If we neglect such disturbing influences as arise from conflict between focal and axial adjustments, and from difference between the optic angle and that between the camera axes when the pictures were taken, and also disregard the fact that the surface of the retina is curved while that of a photograph plate is plane, the following method perhaps will give the best results.

In fig. 2, let C and C′ be the centers of two camera lenses whose principal axes are as usual parallel, and a pair of secondary axes forming an angle, θ, in a horizontal plane, are directed upon an object, A, in the foreground of a scene. Let E be the midpoint between C and C′; then EA is a median; on this prolonged let B be an object in the background. Parallel to C′ C and to the vertical plane of the sensitized plates

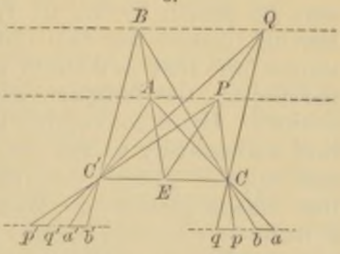


2. 3. 4.

behind the lenses, let two planes be passed through A and B respectively. Let P and Q be any points on these planes, so related that the straight line Q P passes through E. On the plates the stereoscopic displacements of the projections of B from those of A are *a b* and *a′ b′*; and it may be easily shown geometrically that the displacements of those of Q from P are each equal to *a b*. This is not shown in the drawing, but a glance at fig. 5 is sufficient.

Let E be midpoint also between a pair of eyes, R and L, in front of which the conjugate photographs are placed after being inverted, and let rays from them be so deviated by semi-lenses as to make $\alpha=\theta$. If the ratio of L R to C′ C be known, the distances E A_1, E B_1, E P_1 and E Q_1 are determined. In binocular vision the direction of the object seen is always estimated from the position of the combined binocular eye, E, and is coincident with that of the median between the two visual axes, but always somewhere in front.* This is universally true for normal eyes, as may be abundantly learned by experiment, whether the axes be convergent, parallel or divergent, and whether the median be at right angles or oblique to the interocular line, LR. In fig. 3, if E_2A_2 and E_2P_2 represent these medians, we have both direction and distance determined for these foreground points. To the right eye, B_1 (fig. 2) appears beyond and to the right of A_1 at an angular distance determined by the stereoscopic displacement, a_1b_1; to the left eye, beyond and to the left at an equal angular distance; to the binocular eye, E_2 (fig. 3), it is hence homonymously doubled at b'_2b_2. To secure single vision of it, the optic angle must be diminished, and through the rectus muscles this at once suggests to the mind increase of distance, producing at the same moment heteronymous doubling of the foreground point A_2, as in fig. 4. Similar remarks apply to P_2 and Q_2.

5.



If α be less than θ, as is often the case, this fact will cause the observer to estimate A_2 to be more distant than it is represented in the drawing, but by no means necessarily so distant as the actual vertex of α. If α be reduced to zero or become negative the sensation of still further change of muscular tension makes the apparent position of A_2 recede still more, and also that of B_2 in the same proportion; but in no case is this determined by intersection of visual axes except when $\alpha=\theta$. No one can have failed to notice the exaggeration of perspective in some stereoscope pictures, produced by making θ large while α is rendered small or negative by mounting the pair too far apart. This indeed was noticed by Wheatstone,† who approaches very near to the idea of possible optic divergence accompanying the perception of binocular relief, when he says, "but I find that an excellent effect is produced when the axes are nearly parallel by pictures taken at an inclination of 7° or 8°, and even a difference of 16° or 17° has no decidedly bad effect." His preconception that optic convergence, even though slight, is indispensable, prevented his apprehension of more than part of the truth. He states, as a remarkable peculiarity, that "although the optic axes are parallel, or nearly so, the image does not appear to be referred to the distance we should, from this circumstance, suppose it to be, but it is perceived to be much nearer." Such large angles as 17° are sel-

* This Journal, III, vol. i, p. 33 et seq.

† Wheatstone, Physiology of Vision, Phil. Mag., 1852, pp. 513–514.

dom resorted to at present. For taking stereographs of statuary, etc., the lenses of the binocular camera are not often more than 80^{cm} or 100^{cm} apart.

That muscular tension is more important than mere intersection of axes in affecting the judgment of distance and size may be shown by aid of Wheatstone's reflecting stereoscope. Having placed the two outline drawings, each 20^{cm} from its mirror so that a distinct combination is attained by axial parallelism, the judgment of distance is as definite as could be desired. Upon converging the axes strongly and giving attention successively to the two monocular images thus obtained, each appears greatly diminished in comparison with the binocular image just seen. Moreover, at the moment one of them is made an object of special attention, the other grows slightly larger. We have thus images of three apparent sizes, according to the degree of muscular tension with which they are separately regarded, while the visual angle remains constant. The visual axes are converged until their intersection is not more than 5^{cm} or 6^{cm} off, and the illusive impression is that each image is in the direction of its own axis much beyond the intersection. But in fact, being monocular images, the direction of the center of each is that of a secondary axis, the right eye perceiving that on the right, instead of the left. Since the optic center and center of rotation are about $6 \cdot 6^{mm}$ apart, the former being displaced toward the nasal side during the experiment, the two secondary axes meet at a very distant point in the rear. While the distance of the monocular image is indeterminate, it is judged easily enough to be *not* at the vertex of either the apparent or real angle determined by the meeting of axes. The experiment is very striking and is not difficult. We have a binocular image, of little more than natural size, with clear judgment of distance, as the result of axial parallelism; two monocular images, of diminished and separately variable size, with very uncertain judgment of distance, as the result of axial convergence, the principal and secondary axes being subjectively interchanged. The apparent diminution in size of the monocular images may be easily observed by crossing the eyes, while holding in front a card on which a sharply defined outline is drawn. I may discuss this still further in a future paper.

No theory of the stereoscope that includes axial divergence is possible, unless we recognize the subjective combination of the two eyes into a single central binocular eye as the point of origin in all perceptions of direction, distance and form. What is essential for binocular vision is not any particular relation between visual axes but rather superposition of retinal images in the binocular eye. What seemed uppermost in the minds of Wheatstone and Brewster * was superposition of external vir-

* Wheatstone, Physiology of Vision, Phil. Mag., 1852, pp. 243 and 246. Brewster, on New Stereoscopes, Phil. Mag., 1852, pp. 17–26.

tual images by causing rays from two pictures to deviate and appear to come from one central combined external picture. This would exclude the possibility of optic divergence, but seems to be still the most generally accepted theory of the stereoscope. In securing dissimilar pictures of the same object by convergence of camera axes we secure the conditions for the perception of binocular relief by divergence of visual axes.

In the diagram attention is called to the identity in position between the optic center of the binocular eye and the only point through which lines can be drawn in such a way as to cause the stereoscopic displacement to be constant for projections of the points where these lines cut the foreground and background planes. This fact alone is enough to suggest that in vision through the stereoscope the midpoint between the eyes must be the point of origin from which distance and direction are to be perceived. A truth that was first recognized as a physiological necessity is thus confirmed by purely mathematical considerations.

The dissociation between focal and axial adjustments in forced convergence or divergence is at first troublesome and productive of indistinct vision, but this vanishes in great measure after a little practice in ocular gymnastics. If the eyes are comfortable we are apt to forget that the vision is abnormal, and to assume that conditions exist which belong only to normal vision. To ascertain what modifications are imposed by physiological conditions upon the generally accepted mathematical theory of the stereoscope has been the chief object of the present investigation.

New York, Sept. 16, 1881.

For still further discussion, see London Philosophical Magazine, Dec. 1881.